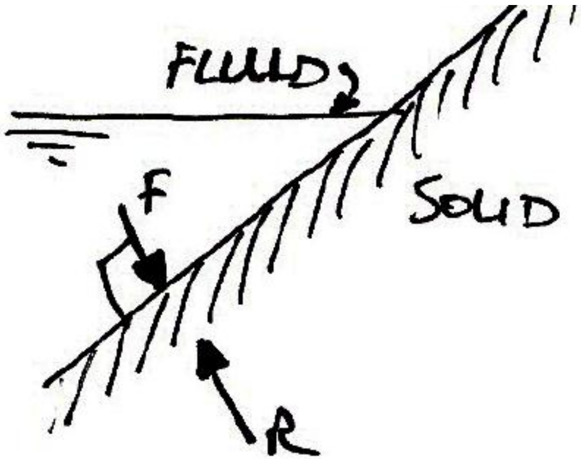
HYDRO
STATICS

PRESSURE IN A FLUID

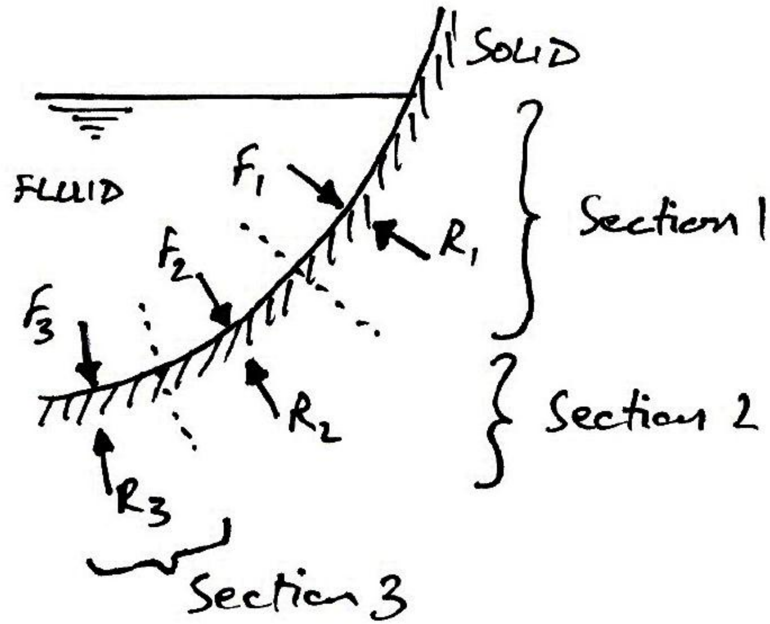
Pressure in a Fluid

Statics of Definition

We applied the definition of a fluid to the static case previously and determined that there must be no shear acting and thus only forces normal to a surface act in a fluid.



Flat surface

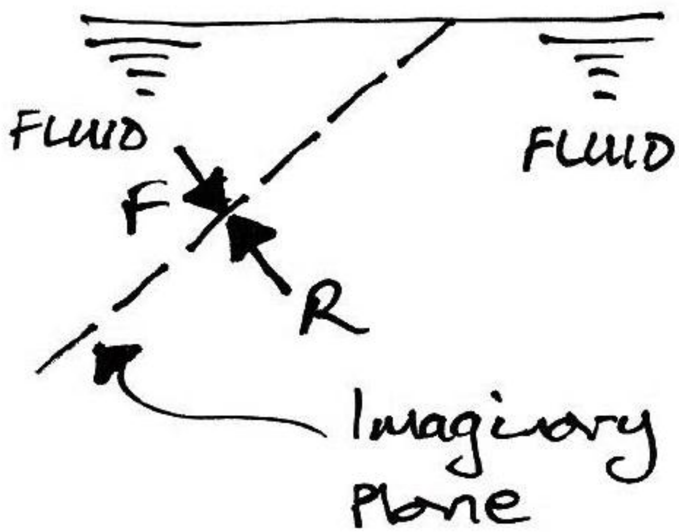


Curved surface

Pressure in a Fluid

Statics of Definition

And we are not restricted to actual solid-fluid interfaces. We can consider imaginary planes through a fluid:



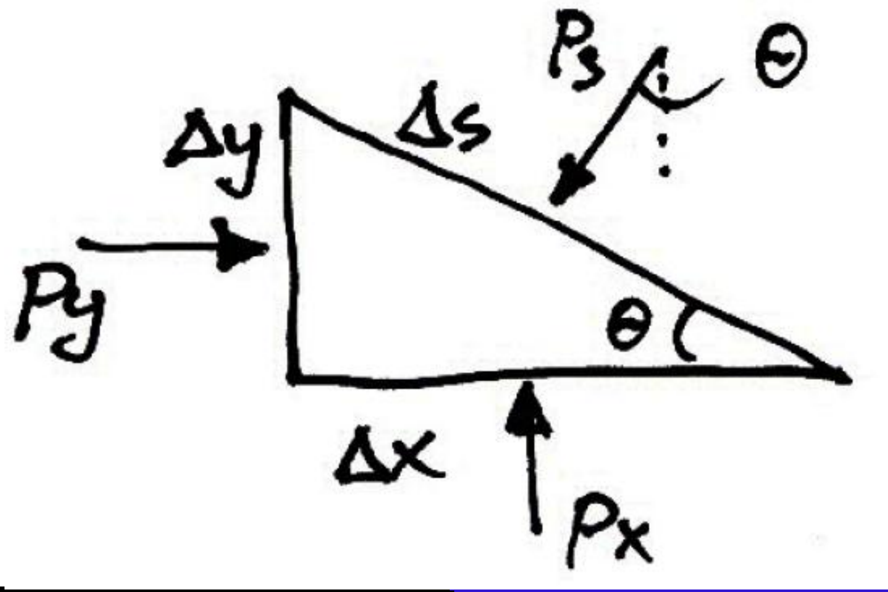
Pressure in a Fluid

Pascal's Law

This law states:

> The pressure at a point in a fluid at rest is the same in all directions.

To show this, we will consider a very small wedge of fluid surrounding the point. This wedge is unit thickness into the page:



Pressure in a Fluid

Pascal's Law

As with all static objects the forces in the x and y directions should balance. Hence:

$$\sum F_x = 0: \qquad p_y \cdot \Delta y - p_s \cdot \Delta s \cdot \sin\theta = 0$$

But $\sin\theta = \dfrac{\Delta \mathrm{y}}{\Delta \mathrm{s}}$, therefore:

$$p_y \cdot \Delta y - p_s \cdot \Delta s \cdot \frac{\Delta y}{\Delta s} = 0$$

$$p_y \cdot \Delta y = p_s \cdot \Delta y$$

$$p_y = p_s$$

Pressure in a Fluid

Pascal's Law

$$\sum F_y = 0: \qquad p_x \cdot \Delta x - p_s \cdot \Delta s \cdot \cos\theta = 0$$

But $\cos\theta = \dfrac{\Delta \text{x}}{\Delta \text{s}}$, therefore:

$$p_x \cdot \Delta x - p_s \cdot \Delta s \cdot \frac{\Delta x}{\Delta s} = 0$$

$$p_x \cdot \Delta x = p_s \cdot \Delta x$$

$$p_x = p_s$$

Hence for any angle:

$$p_y = p_x = p_s$$

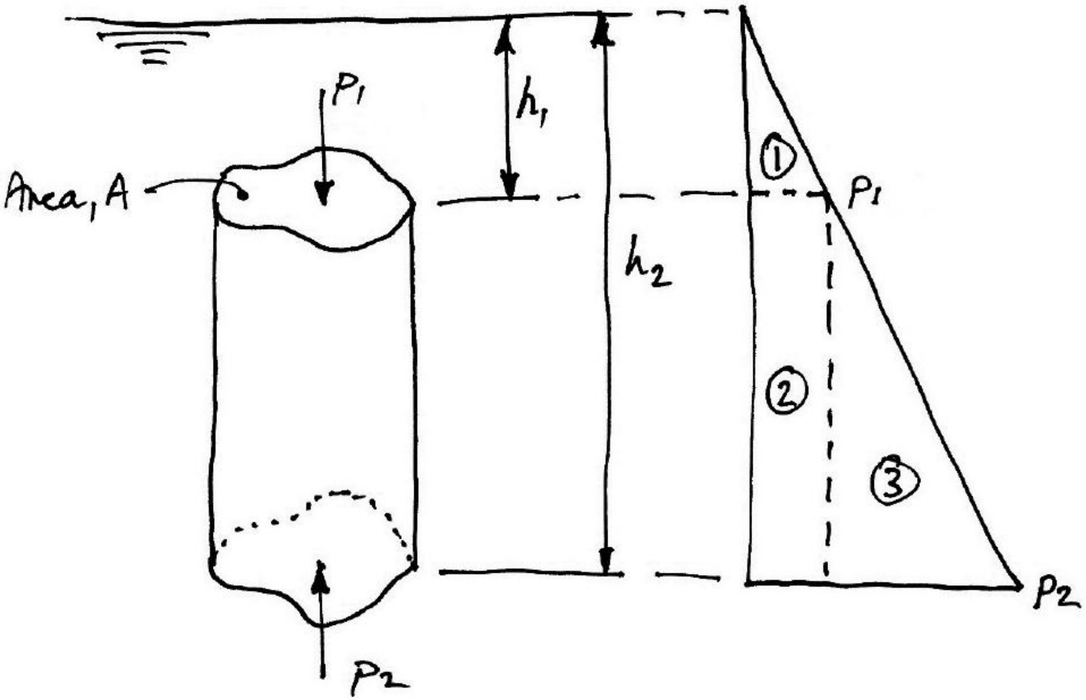
And so the pressure at a point is the same in any direction. Note that we neglected the weight of the small wedge of fluid because it is infinitesimally small. This is why Pascal's Law is restricted to the pressure at a point.

Pressure in a Fluid

Pressure Variation with Depth

Pressure in a static fluid does not change in the horizontal direction as the horizontal forces balance each other out. However, pressure in a static fluid does change with depth, due to the extra weight of fluid on top of a layer as we move downwards.

Consider a column of fluid of arbitrary cross section of area, A:



Column of Fluid

Pressure Diagram

Pressure in a Fluid

Pressure Variation with Depth

Considering the weight of the column of water, we have:

$$\sum F_y = 0: \qquad p_1 A + \gamma A (h_2 - h_1) - p_2 A = 0$$

Obviously the area of the column cancels out: we can just consider pressures. If we say the height of the column is $h = h_2 - h_1$ and substitute in for the specific weight, we see the difference in pressure from bottom to the top of the column is:

$$p_2 - p_1 = \rho g h$$

This difference in pressure varies linearly in h, as shown by the Area 3 of the pressure diagram. If we let $h_1 = 0$ and consider a gauge pressure, then $p_1 = 0$ and we have:

$$p_2 = \rho g h$$

Pressure in a Fluid

Pressure Variation with Depth

Where h remains the height of the column. For the fluid on top of the column, this is the source of p_1 and is shown as Area 1 of the pressure diagram. Area 2 of the pressure diagram is this same pressure carried downwards, to which is added more pressure due to the extra fluid.

To summarize:

The gauge pressure at any depth from the surface of a fluid is:

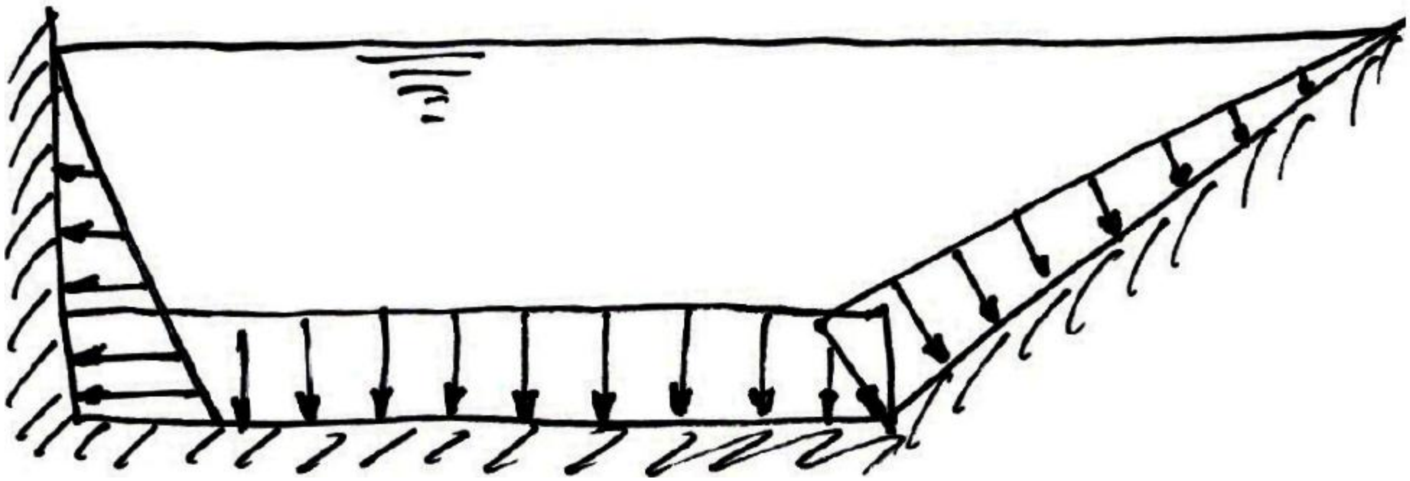
$$p = \rho g h$$

Pressure in a Fluid

Summary

1. Pressure acts normal to any surface in a static fluid;
2. Pressure is the same at a point in a fluid and acts in all directions;
3. Pressure varies linearly with depth in a fluid.

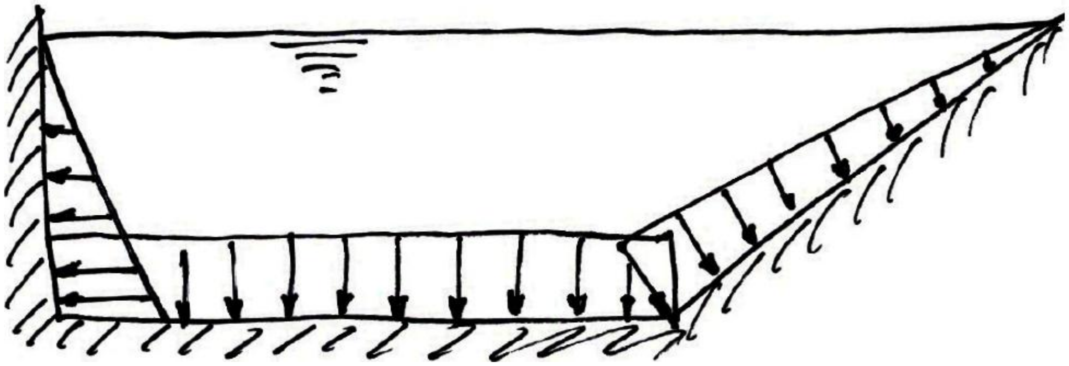
By applying these rules to a simple swimming pool, the pressure distribution around the edges is as shown:



Pressure in a Fluid

Note:

1. Along the bottom the pressure is constant due to a constant depth;
2. Along the vertical wall the pressure varies linearly with depth and acts in the horizontal direction;
3. Along the sloped wall the pressure again varies linearly with depth but also acts normal to the surface;
4. At the junctions of the walls and the bottom the pressure is the same.



Pressure in a Fluid

Problem Set 2

Problem 1

If a depth of liquid of 1 m causes a pressure of 7 kPa, what is the specific gravity of the liquid?

Ans: s = 0.714

Pressure in a Fluid

Problem Set 2

Problem 2

What is the pressure 12.5 m below the ocean? Use specific gravity of 1.03 for salt water.

Ans: p = 126.3 kPa

Pressure in a Fluid

Problem Set 2

Problem 3

If the pressure in the air space above an oil (s = 0.75) surface in a closed tank is 115 kPa absolute, what is the gauge pressure 2 m below the surface?

Ans: p = 28.39 kPa

Pressure in a Fluid

Problem Set 2

Problem 4

Find the absolute pressure in *kPa* at a depth of 10 *m* below the free surface of oil (s = 0.75) if the barometric reading is 752 *mm* Hg.

Ans: p_{abs} = 173.9 *kPa*

Pressure in a Fluid

Problem Set 2

Problem 5

A pressure gage 6 m above the bottom of the tank containing a liquid reads 90 kPa. Another gage of height 4 m reads 103 kPa. Determine the specific weight of the liquid.

Ans: y = 6.5 kN/ m^3

Pressure in a Fluid

Problem Set 2

Problem 6

An open tank contains 5.8 m of water covered with 3.2 m of kerosene ($y = 8\ kN/m^3$). Find the pressure at the interface and at the bottom of the tank.

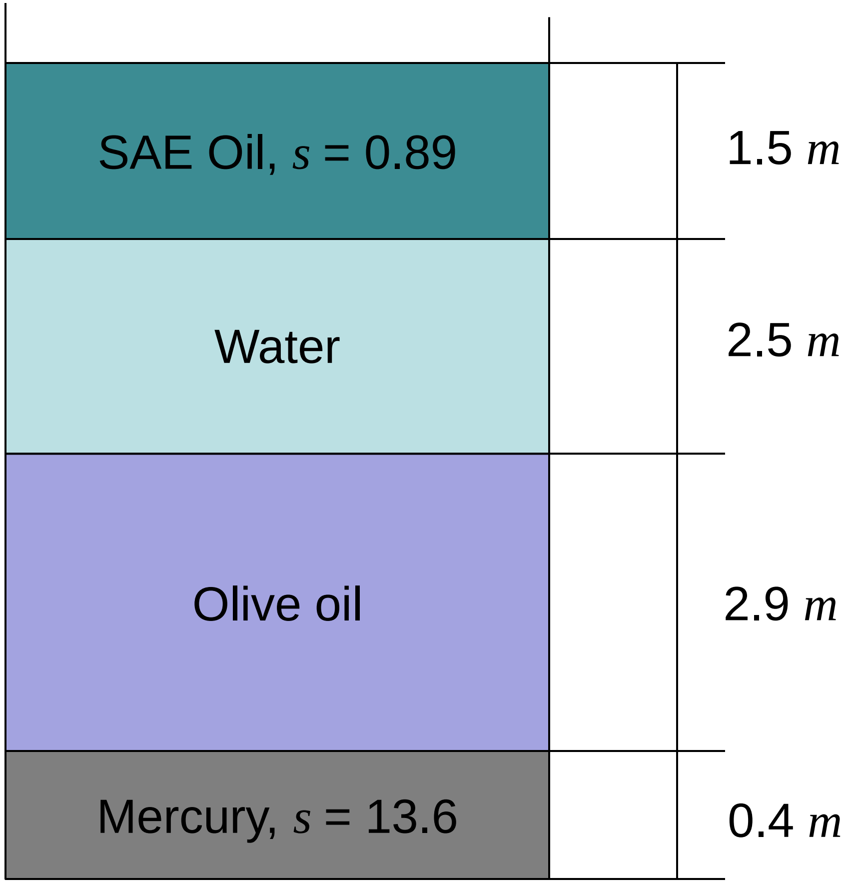
Ans: $p_A = 25.6\ kPa$, $p_B = 82.498\ kPa$

Pressure in a Fluid

Problem Set 2

Problem 7

In the figure shown, if the atmospheric pressure is 101.03 kPa and the absolute pressure at the bottom of the tank is 231.3 kPa, what is the specific gravity of olive oil?



Ans: s = 1.38

Pressure in a Fluid

Problem Set 2

Problem 8

A hydraulic press is used to raise an 80-kN cargo truck. If oil (s = 0.82) acts on the piston under a pressure of 10 Mpa, what diameter of piston is required?

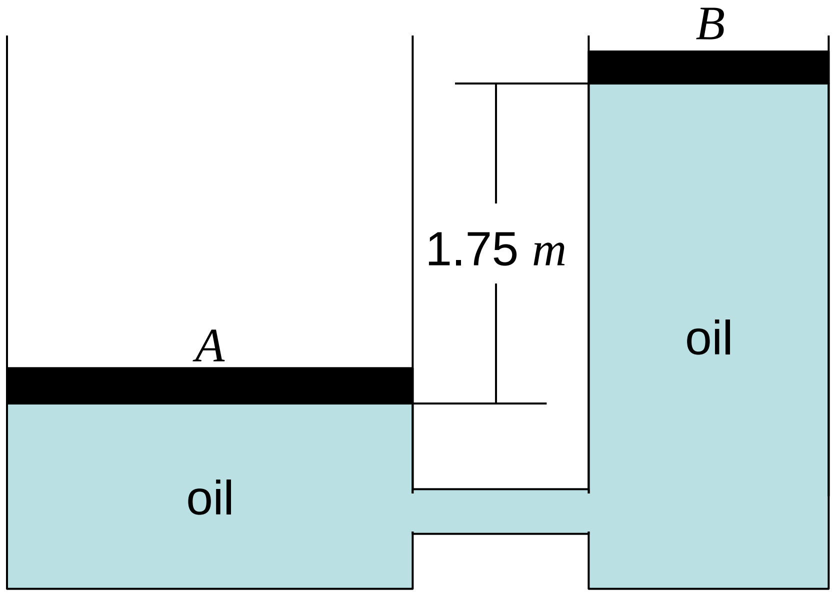
Ans: D = 100 mm

Pressure in a Fluid

Problem Set 2

Problem 9

Piston A has a cross-section of 1,200 cm^2 while that of piston B is 950 cm^2 with the latter higher than piston A by 1.75 m. If the intervening passages are filled with oil (s = 0.8), what is the difference in pressure between A and B?



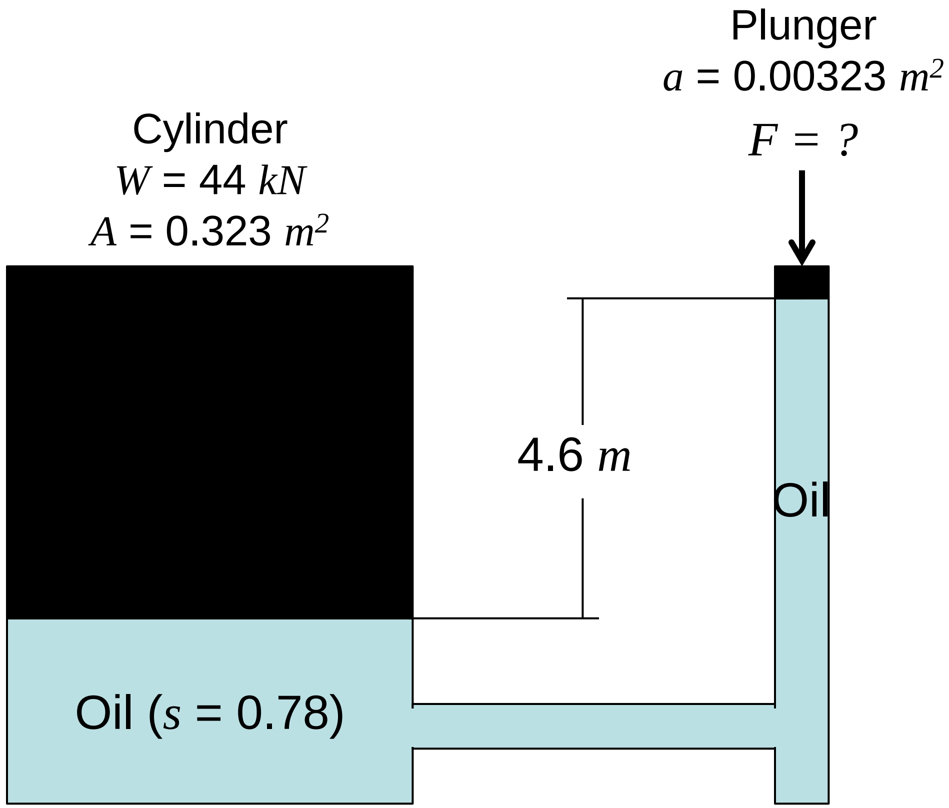
Ans: $p_A - p_B = 13.734\ kPa$

Pressure in a Fluid

Problem Set 2

Problem 10

The figure shown shows a setup with a vessel containing a plunger and a cylinder. What force F is required to balance the weight of the cylinder if the weight of the plunger is negligible?



Ans: $F = 0.326\ kN$